Ex How much Error is there for the points $(1,3), (2,6), (3,8), (4,6)$ fitted with the line $y = 1.1x + 3$?

	x	$y = 1.1x + 3$	E_i
$(1,3)$	1	4.1	$4.1 - 3 = 1.1$
$(2,6)$	2	5.2	$5.2 - 6 = -0.8$
$(3,8)$	3	6.3	$6.3 - 8 = -1.7$
$(4,6)$	4	7.4	$7.4 - 6 = 1.4$

$E = 1.1^2 + (-0.8)^2 + (-1.7)^2 + (1.4)^2$

~~6.6664~~

$= 6.7$

How do we minimize error?

Can we use a different line?

$$y = Ax + B$$

Error only depends on A and B

can use optimization with partial derivatives.

ex Find the straight line that minimizes the error for the points $(1,4), (2,5), (3,8)$

$$E_1 = A\cdot(1) + B - 4$$
$$E_2 = A\cdot(2) + B - 5$$
$$E_3 = A\cdot(3) + B - 8$$

Total error $$E = (A+B-4)^2 + (2A+B-5)^2 + (3A+B-8)^2$$

E is a function of A & B.

want to find (A,B) that minimizes $E(A,B)$

$$\frac{\partial E}{\partial A} = 2(A+B-4) + 2(2A+B-5)\cdot 2 + 2(3A+B-8)\cdot 3$$
$$= 28A + 12B - 76$$

$$\frac{\partial E}{\partial B} = 2(A+B-4) + 2(2A+B-5) + 2(3A+B-8)$$
$$= 12A + 6B - 34$$

$$\begin{cases} 28A + 12B - 76 = 0 \\ 12A + 6B - 34 = 0 \end{cases}$$

$$\begin{cases} 28A + 12B = 76 \\ 12A + 6B = 34 \end{cases}$$ divide top by 2

$$\begin{cases} 14A + 6B = 38 \\ 12A + 6B = 34 \end{cases}$$ subtract 2nd from 1st

$$2A = 4 \Rightarrow \boxed{A = 2}$$

$$12(2) + 6B = 34$$

$$24 + 6B = 34$$

$$6B = 10 \Rightarrow \boxed{B = \frac{5}{3}}$$

So $y = 2x + \frac{5}{3}$ minimizes error.

ex Given points $(1,9), (2,8), (3,6), (4,3)$

which line below fits best?

(has smallest error)

A) $y = -2x + 12$

B) $y = -2x + 11$

	$(1,9)$	$(2,8)$	$(3,6)$	$(4,3)$	Total Error
$y=-2x+12$	$(1,10)$	$(2,8)$	$(3,6)$	$(4,4)$	$1^2+0^2+0^2+1^2 = 2$
$y=-2x+11$	$(1,9)$	$(2,7)$	$(3,5)$	$(4,3)$	$0^2+1^2+1^2+0 = 2$

Both have same error.

Ex U.S. Per Capita Health Care Expense

Data:

Years (After 2000)	Dollars
9	8175
10	8428
12	8996
13	9255

$$E = (9A+B-8175)^2 + (10A+B-8428)^2 + (12A+B-8996)^2 + (13A+B-9255)^2$$

$$E_A = 18(9A+B-8175) + 20(10A+B-8428) + 24(12A+B-8996) + 26(13A+B-9255)$$

$$E_B = 2(9A+B-8175) + 2(10A+B-8428) + 2(12A+B-8996) + 2(13A+B-9255)$$

E_A: $988A + 88B = 772244$

E_B: $88A + 8B = 69708$

$$A = \frac{1364}{5}, \quad B = \frac{57127}{10}$$

so $y = \frac{1364}{5}x + \frac{57127}{10}$ is line of Best fit

$$y = 272.8x + 5712.7$$

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Both have same error.